

HISTORY, DIFFERENTIAL EQUATIONS, AND THE PROBLEM
OF NARRATION

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The talk of engineers and the talk of historians connects in the way that metaphors connect with stories. Though both make use of both (because we cannot live otherwise in the world), engineers specialize in metaphors and historians in stories.

In the abstract it is a matter of definitions. Take the essence of the metaphor to be comparison and the essence of the story to be time. (And let the definitions ride; they break down soon enough if they are wrong.) A comparison can be put as a timeless law, such as that apples and the moon fall to earth or that imperial hubris usually gets punished. The moon is like an apple; Athens in the Peloponnesian War was like a proud man. A history, on the other hand, can be stated as a sequence of connected events. The moon was here on January 1, and therefore will be just so on April 2, eclipsing the sun; Athens overreached to Sicily in the sixteenth summer, and therefore only a few returned out of many.

The engineer and historian do not deal in mere comparison or mere time, no more than poets or novelists do. Aimless comparison is bad poetry and bad engineering; one damned thing after another is bad fiction and bad history. The point is pointedness, which will vary with the purpose in mind. (The point, incidentally, need not be simple or realistic or of any other kind especially approved by the nineteenth century.) Comparing a pendulum to the wavering of the beloved's affection may have a point in a certain poem, but to have a point as engineering the pendulum needs to be compared with, say, an ideal falling body constrained by an ideal shaft. A bare chronology likewise makes no history, unless pointed by the context and the questions. The dated sequence of winning numbers on the Iowa lottery from 1987 to 1990 is a chronology, but can be read as history only in the imaginings of an numerologist.

Metaphor and story, then, are two ways of arguing. Place pure metaphor at one end, containing no element of story, "timeless" as I have said. I shall compare thee to any old summer's day, not to the remarkable events of May 21-28, 1595. At the other end place a mere listing of the events in order: "May 23, midday, rough winds; at sunset, gold complexion dimmed."

The two ends are linked by a theme. That is the first point here. A list of events in World War I or in the market for hogs is not much of a story; maybe it is not a story at all, a sheer chronicle. But when thematized — by "God favors big battalions," for instance — the story can be told persuasively. Letting in a trace

of metaphor gives shape to the chronicle and pushes it towards a good story. It works from the other end, too. That God favors big battalions is a pure metaphor, and rather lacking in point. When clothed in time it begins to look more story-like. Themes can be good or bad. God may favor the big battalions only in the very long run, over a century of battle, say. Such a slow-working model will be a poor theme in a story of World War I. To tell the story we will have to seek some other thematizing commonplace, such as that God favors the smarter generals or that God favors the British Empire or, as many of the combatants came to believe, that God did not give a damn after all.

A thematized story, or a dynamized model, stands between the pure (and mere) metaphor and the pure (and mere) story. The good middle ground seems to be an allegory, which persuades on both grounds: the Christian life is like a journey, and Christian's actual journey in time to the Heavenly City brings the metaphors to a life. Thus, a metaphor that people are calculating machines, when applied to the market for wheat, might lead an economist to a little equation that speaks of time: the rate of change of price, she says, is equal to some number multiplied by the gap between the actual and the long-run price. The number tells how quickly the market responds. It is a thematized story, most useful in life. How sluggish is the market for hogs? A large number in the economist's equation will have the market leaping smartly to it; a small one and the market lingers abed.

Again, the physicist's metaphor of the point mass falling under gravity leads to that splendid theme, Force equals mass times acceleration, etched on the shields of physicists. When applied to a pendulum, $F = ma$, itself a metaphor (though speaking quietly about time in mentioning acceleration), leads to a theme still more explicitly historical: how the next step in the history of a swinging pendulum will depend on gravity and the resistance of the fluid in which it swings. The new equation is closer to history. The equation is story-like because it speaks of time and therefore organizes experience in time, at least implicitly. The time-speaking themes will shape the raw experience, as a story does when it is more than a mere unthematized chronicle.

Such themes in engineering that quietly mention time carry the imposing title of Differential Equations. Not all differential equations need speak of time, but many in engineering and physics and economics do. They say, "The rate of change of X depends on various matters, among them the time that the situation has been developing." A differential equation involving time is in a sense neither fish nor fowl, neither a timeless metaphor nor a pure story. Notions such as that people are calculating machines or that the pendulum is rigidly fixed are the timeless metaphors, true at all times and therefore not making reference to time ($F = ma$, to repeat the example, is a metaphor already on the way to a story, since it speaks of speeding up per unit of time, the acceleration). The differential equation that is derived from the metaphors is the middle type, a link between pure metaphors and pure stories. It is an allegory with inexplicit meaning; the analyst makes it explicit.

The idea of a differential equation can be grasped simply, although whole libraries filled with mathematics have been devoted to the complications. An ex-

ample of a differential equation is: "The world's population is increasing by 80 million people a year." It gives a thematized story about how something will change. Population is said to be just like a series of numbers that rise by 80 million in each year. Time is mentioned, quietly—the equation has no dates in it, but the dates are implicit. In 1991 the change will be 80 million people, in 1992 another 80 million, and so forth. (Needless to say the differential equations in the libraries are not so simple. And in fact this particular one, though popular in the newspapers, is not very sensible as demography. It says that population will go on growing, like a frictionless pendulum in a vacuum will go on swinging, despite forces to the contrary.)

Algebraically speaking, the differential equation is that the population next year, or time $t+1$, is the population now, time t , plus 80 million: $P_{t+1} = P_t + 80$ million. To put it another way, the way that differential equations are in fact often put, the *change* in P (called " dP ") per year (that is, divided by " dt ," the change in time) is 80 million. As the mathematicians have said since Leibniz, $dP/dt = 80$ million.

The time path of population is going to be whatever it is in 1990 (called in the trade "the initial conditions") plus 80 million, then plus 80 million, and so forth. The time path is the pure story, the mere chronicle unthematized: the world in 1990 has 4,500 million, the world in 1991 will have 4,580 million, the world in 1992 will have 4,660 million, and so forth into an ecological disaster or a flowering of world civilization, depending on one's other beliefs. Without a theme the set of numbers is just one damned thing after another, the purest story.

The shape of the chronology is more apparent in what is known officially as the "solution" to the differential equation. The solution characterizes the chronology. In this case, obviously (as the mathematician in the joke said about the step that had taken him six months of contemplation to believe), the solution is: the population in a year that is t years after the initial year is the initial population plus t times 80 million. That is, $P_t = P_0 + t(80 \text{ million})$.

None of this is higher math, but it illustrates the essence of some very high math indeed. You can always simply mount the differential equation on a computer, start somewhere, and then grind out a future. Start at 4,500 million people, note that the differential equation says that the annual change is 80 million, and then calculate successively 4,580 million then 4,660 million and so forth. The set of numbers about the future is the brute chronology, which you can look at and perhaps impose interesting themes upon.

When differential equations have solutions (and the first surprising fact about the higher math is that it is very easy to write down ones that do not) the themes are fully explicit. Just to show that the "solutions" are by no means always as trivial as the example of population change, think of a pendulum swinging in a fluid. An engineer might come to think about the angle ϕ that a moderately swinging pendulum (as in a clock) deviates from the straight up and down at any time t . Suppose he signifies the rate of change of ϕ as ϕ' (note the prime: it signals the "first derivative" of ϕ with respect to time). Consequently the acceleration rate of change of the rate of change, or the second derivative, will be called

ϕ'' . If he thinks extremely perspicaciously about the situation it turns out that he will come to believe that the angle ϕ must satisfy always the differential equation:

$$* \quad \phi'' + k_1\phi' + k_2\phi = 0.$$

(The k 's are constants related to the force of gravity and the viscosity of the fluid, such as air, in which the pendulum swings.) If he has gotten this far by unaided intellect he is probably the great Swiss mathematician Leonhard Euler (1707–1783) and can write down directly the astounding solution, that is to say, an equation that gives you the time path of the pendulum for every future (or past) time, t . It is the theme in the motion for ϕ as time goes by. It involves sines and cosines, those high school horrors, and a number e , about 2.71828, which Euler showed keeps coming up in mathematics:

$$\text{Solution to } *: \phi = (c_1 e^{bt} \sin at) + (c_2 e^{bt} \cos at).$$

Like the terms of most human histories, most solutions of differential equations in this explicit form (called “analytic solutions”) cannot be achieved mechanically. They have to be guessed at, then confirmed by showing they correspond with the original equations, which is to say with the partially thematized chronologies that we call history. Even the ones that do not have analytic solutions often have approximate solutions in terms of what are called, alarmingly, “infinite series.” The successive terms of such series are approximate themes. For instance, the large first term in an infinite series of themes for World War I might be “God favors the bigger battalions,” to which might be added the somewhat less important second term (“... and the better generals”), to which might be added the third (“... and the British Empire”), and so on, out to the limit of the historian’s or the engineer’s need for thematization. (The engineer uses the thematization to characterize and predict, the historian to characterize and explain, but otherwise they are doing much the same job: one is predicting, the other postdicting.)

So the solution to the differential equation $*$ is a more direct and flat-footed story, shaping experience more explicitly, than the differential equation itself. And finally a particular, numerical history derived from either the differential equation or its solution—such as “when time is at 1.0 the pendulum is at $\phi = +15^\circ$, when time is at 1.1 it is at 12° ,” and so forth—is the most flatfooted and chronological of all, the Cliff Notes, so to speak, laying out the plot of *The Mayor of Casterbridge*.

The differential equation itself might be looked on as the model/metaphor. Alternatively, and I think better, the honor of the word “metaphor” might be reserved for the timeless physical or economic or historical idea behind the equation, such as that the moon falls towards the earth or that people pursue profits in buying wheat or that the men of Athens were very fools in their imperial might. The actual numerical time paths from the solution of a differential equation is the narration in time, and the solution (which can also generate the numerical path) is the thematized narrative—transparent or muddy depending on how neat

the solution is, when it exists. The analytic solutions correspond to simply predictable histories, that is, histories that can be reexpressed as equations. The differential equations embody what we think we know about societies as theory, such as a Marxist theory. The solution then characterizes a particular historical path. If the path can be expressed in terms of what the mathematicians call elementary functions, then what is being asserted is that history is in every sense elementary. The law of surplus value leads, the Marxist historian/engineer asserts, to a falling rate of profit and a crisis of capitalism, as the night will follow day.

Such talk undermines the claim that natural science and historical science have two separate modes of apprehension. The separation seems less consequential if it is viewed merely as the metaphor as against the story, and if in good metaphors and good stories the two are linked by a differential equation. The old question — Clio, Science or Muse? — loses its gripping interest if sciences use stories and art uses number.

But why bother with all this? All right, history and engineering can be made both to lie down on a Procrustean bed of the differential equation. But why on earth would one wish to speak in such a way?

The answer is this: speaking so makes it possible for each of the two modes of thinking to criticize the other. "Criticize," I say, not "lambast" or "sneer at" or even "blindly worship," which are some of the present ways the two cultures deal with each other. I am arguing that there is a similarity between the most technical scientific reasoning and the most humanistic literary reasoning.

I have given some economic examples in *The Rhetoric of Economics* (1985) and *If You're So Smart* (1990). Here I want to give another example, of how life gets difficult for the engineer and the historian when the differential equation does.

The big difficulty shows up when the differential equations are "nonlinear." The word means simply that the variables described by the equation and their "derivatives," the rates of change of the variables, are squared or sined or in some other way blown up beyond their simple, linear selves, plain ϕ or ϕ' or ϕ'' . The change turns out to be important. As a good textbook on differential equations put it in 1972, "The theory of *linear* differential equations has been studied deeply and extensively for the past 200 years, and is a fairly complete and well-rounded body of knowledge. However, very little of a general nature is known about *non-linear* equations."¹ A lot has been learned since 1972, some of it under the name of "chaos."² One gathers that a lot remains to be learned.

The commonest theme of battle history, the horseshoe nail, is a case of a non-linear differential equation: For want of a nail the shoe was lost. / For want of the shoe the horse was lost. / For want of the horse the rider was lost. / For want of the rider the battle was lost. / For want of the battle the kingdom was lost. / And all for the want of a horseshoe nail. The rate of loss feeds on itself.

1. George F. Simmons, *Differential Equations, with Applications and Historical Notes* (New York, 1972), 290, italics added.

2. See James Gleick, *Chaos: Making a New Science* (New York, 1987), an enthralling popular history of the recent work.

Battle-history is not held in high regard by historians precisely because it so obviously depends on tiny chances of this sort. The popular history magazine *American Heritage* will turn to them when explaining the battle of Chancellorsville: "Only the failure of one inept Confederate officer saved the Federal army from unmitigated disaster."³

But the disdain for assigning large events small causes is not rational in a world partly nonlinear. A recent book by the economist and historian Robert Fogel argues that there was nothing inevitable about Lincoln's election and the resulting secession. Like many historians before him he emphasizes the precarious balance of American politics in the 1850s, which could have been turned one way or the other by minor events. In the late 1850s:

The Republican party was not wrecked by the panic of 1857 and by 1860 it had lured most of the former Know-Nothings into its ranks. However, neither outcome was inevitable. . . . It is doubtful that party leaders could have continued to suppress the nativist impulses of so many of its members if immigration had returned to the 1854 rate. . . . If the party would have conceded these demands, some of the Germans and the more conservative Whigs would have been alienated. Only relatively small defections were needed to deny power to the anti-slavery coalition in 1860.⁴

And during the fateful month of May, 1856 in bloody Kansas:

a sheriff who had proprietary interests in a rival town not far from Lawrence, and who was an impulsive extremist, took unauthorized command of the posse. The mob that he led burned the hotel that served as the headquarters for the New England Emigrant Aid Society. . . . Two days later, in retaliation for the "sack of Lawrence," John Brown and his sons killed "five helpless and unprepared pro-slavery settlers." . . . As the posse moved toward Lawrence, Senator Charles Sumner (R, MA) delivered a searing indictment. . . . of leading Democratic members of the Senate, including Stephen A. Douglas (D, IL) and Andrew P. Butler (D, SC). Butler was absent from the chamber during Sumner's speech but Preston S. Brooks, a relative and a member of the House from South Carolina, brooded over the insults to his aged kinsmen and to his state. . . . Brooks entered the Senate chamber after it adjourned on May 22 and delivered a series of blows to Sumner's head and shoulders with his cane.⁵

Fogel identifies other turning points, too. He is trying to show that the end of slavery was by no means determined by massive and unstoppable forces, such as its alleged unprofitability or its alleged inconsistency with industrialization. "The overarching role of contingent circumstances in [the] ultimate victory [of the antislavery movement] needs to be emphasized. There never was a moment between 1854 and 1860 in which the triumph of the antislavery coalition was assured."⁶

James McPherson's recent history of the same era provides military examples. "The third critical point came in the summer and fall of 1863 when Gettysburg,

3. Robert K. Krick, "Lee's Greatest Victory," *American Heritage* 41 (March, 1990), 74.

4. Robert Fogel, *Without Consent or Contract: The Rise and Fall of American Slavery* (Boston, 1989), 385-386.

5. *Ibid.*, 379.

6. *Ibid.*, 322.

Vicksburg, and Chattanooga turned the tide toward ultimate northern victory."⁷ Vicksburg was settled by many things — one is put in mind of the much-abused term "over-determination" — but among them was a disagreement before the siege between the Confederate generals Joe Johnston and John C. Pemberton.

Johnston urged Pemberton to unite his troops with Johnston's 6,000 survivors north of Jackson [Mississippi], where with expected reinforcements they would be strong enough to attack Grant. . . . Pemberton disagreed. He had orders to hold Vicksburg and he intended to do so. . . . Before the two southern generals could agree on a plan, the Yankees made the matter moot by slicing up Pemberton's mobile force on May 16 at Champion's Hill.⁸

At Gettysburg one of numerous turning points was the desperate defense of Little Round Top on July 2 by Colonel Joshua L. Chamberlain of the twentieth Maine. Chamberlain (who not incidentally was in civilian life a professor of rhetoric) ordered his men, ammunition exhausted, to attack with bayonets the massing Confederates down the hill. "(T)he two Round Tops dominated the south end of Cemetery Ridge. If the rebels had gotten artillery up there, they could have enfiladed the Union left. . . . (S)hocked by the audacity of this bayonet assault, the Alabamians surrendered by scores to the jubilant boys from Maine."⁹

In Michael Shaara's fine historical novel about Gettysburg, *The Killer Angels*, one can list the contingencies on which the battle depended: John Buford's eye for good ground on which the Federals could stand; the bullet that had already killed Stonewall Jackson; Richard Ewell's hesitation, which kept the Confederates from taking Cemetery Ridge before it was fortified; Lee's heart disease slowing his decisions; Jeb Stuart's failure to stay in touch with Lee's headquarters; Longstreet's inability to persuade Lee to move left; Longstreet's decision not to disobey Lee; and so forth.¹⁰ Only a novelist's touches? They had better be an historian's too, if he wants to tell the story.

In the conclusion to his book McPherson writes, "Northern victory and southern defeat in the war cannot be understood apart from the contingency that hung over every campaign, every battle, every election, every decision during the war. This phenomenon of contingency can best be presented in a narrative format."¹¹ Precisely. Fogel and McPherson and Shaara are telling the usual horseshoe-nail story.

Little events can have big consequences in some parts of history. The parts are described by models that are nonlinear in the events and whose consequences feed on themselves. In other words, a little event affecting one of the equations yields a large consequence, which is then fed back as input. "Nothing succeeds like success" is such a model, and certainly applies to the decade 1856–1865 in the United States. It was the strategy of both the Union and Confederacy for much of the war to win one more battle in order to bring Britain or France in on the correct side.

7. James M. McPherson, *The Battle Cry of Freedom: The Civil War Era* (New York, 1988), 858.

8. *Ibid.*, 630.

9. *Ibid.*, 659.

10. Michael Shaara, *The Killer Angels* (New York, 1974).

11. McPherson, 858.

The point is not that great oaks from little acorns grow. They do, as did Christianity and the Industrial Revolution. The right acorn is impossible to see before the event. The historical economist Joel Mokyr identifies a common pitfall in storytelling: rummaging among the possible acorns from which the great oak of the Industrial Revolution grew "is a bit like studying the history of Jewish dissenters between 50 B.C. and 50 A.D. What we are looking at is the inception of something which was at first insignificant and even bizarre," albeit "destined to change the life of every man and woman in the West."¹² Any one of numberless acorns may be chosen by chance. Chance of this unconventional sort is similarly difficult to narrate. But at least after the acorn is chosen it grows slowly from acorn to sprout to sapling to tree, shaped by the great forces of its environment and at each stage more prominent. Mokyr misspeaks in saying that the party of Jesus was "destined" to win. Much had to happen before it did.

The point here is rather that in some modeled worlds an acorn produces by itself a great tree in an instant. Such a world is unstable, as in the world of the United States in 1856-1865. The models need not be complicated. As students of chaos theory since Poincaré have pointed out, simple models can generate astonishingly complicated patterns. The slightest perturbation can yield an entirely different history. (And in catastrophe theories, quickly.) Confederate success depended on recognition by Great Britain, which depended on . . . Confederate success. It depended on, among other things, human wills at Lawrence, Kansas or Little Round Top.

The basic model is $x_{t+1} = f(x_t)$, a one-period difference equation, where the function f is nonlinear (for example, squared). The simplest equation is the hump: $x_{t+1} = \beta x_t [1 - x_t]$, where β is the so-called "tuning parameter" to give some variation in how the hump looks. It might tell you how the population of rabbits in year $t+1$ depends on the population in year t (considering that wolves, who eat rabbits, depend, too, on how many rabbits there are). Look at Figure 1.¹³ The 45° line serves merely to translate x_{t+1} back into a new x_t to get stuck into the equation again for another iteration. A few iterations are shown, starting at point A in the year- t rabbit population. The rule is: all horizontal moves are to the 45° line, all vertical ones back to the humped curve. Just follow out the sequence vertical-horizontal-vertical-horizontal and so forth, forever. The succession of value below the emphasized dots are how the rabbit population changes, the chronological history deriving from the differential equation.

Written out the equation is $x_{t+1} = \beta x_t - \beta x_t^2$, the squared term signaling the nonlinearity. The equation says that the next value of the variable x (which can be the population of rabbits or the offer at the arms control negotiations) is proportional to the value now, but gets driven down (by the squared term) if it gets too high. Too many rabbits will give scope for too many wolves, with a consequent crash in rabbit populations. The humped curve is the equation; the sharper

12. Joel Mokyr, *The Economics of the Industrial Revolution* (Totowa, N.J., 1985), 44.

13. From William Baumol and Jess Benhabib, "Chaos: Significance, Mechanism, and Economic Applications," *Journal of Economic Perspectives* 3 (Winter, 1989), 82.

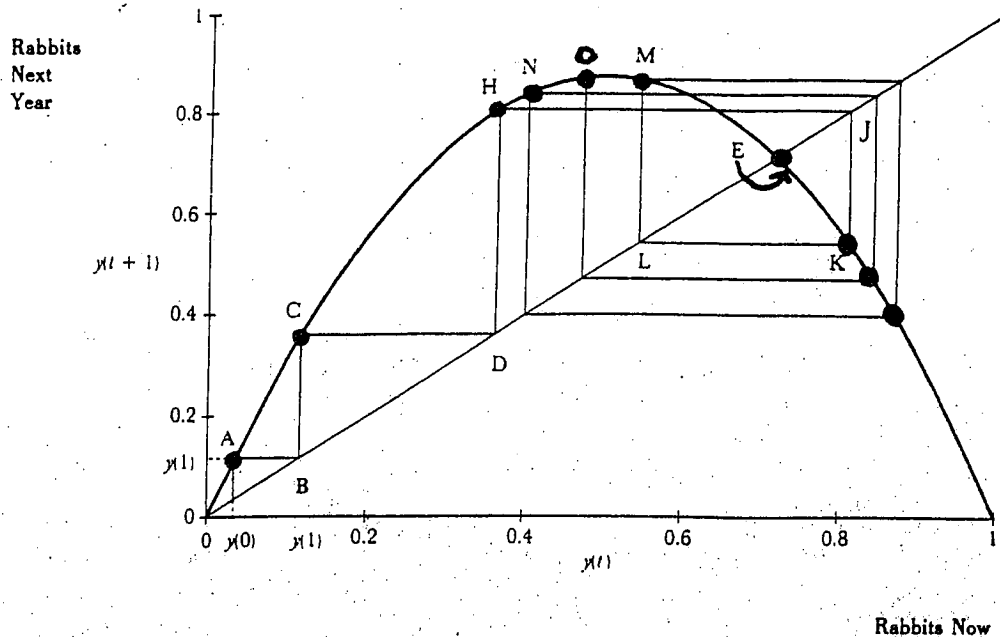
Fig. 1. Phase diagram, periods 0-9 $x(t+1) = 3.5x(t)[1 - x(t)]$, $x(0) = 0.034$ 

Figure 1:
How a Population Changes According to
a Nonlinear Differential Equation

is the hump the higher is β . The more humped the more violently nonlinear is the differential equation. Notice that by first-year calculus the rate of change of x , dx_{t+1}/dx_t , is just $\beta(1 - 2x_t)$, β being the violence number. Notice too that the mathematics is not exactly "feeding on itself": it is more like gnawing on itself, reducing its rate of change as it (whatever it is) gets larger.

Look at Figure 2.¹⁴ It consists of the plots of the first fifty iterations on the same equation, with β at 3.94 in the top one and only slightly different, 3.935, in the bottom. At first the two look alike. But they soon diverge: look for example at the strange stability from point B to point C in the bottom plot.

The jargon is "sensitive dependence on initial conditions." In a nonlinear world the history depends sensitively on where you start. A point A' a little bit away from A would yield eventually an entirely different time path of rabbits. Another nice bit of jargon is the "average rate of local trajectory spreading," which is to say how fast two points originally close to each other split apart. A world in 1863 without a Colonel Joshua Lawrence Chamberlain is easy to imagine, since it is close to the actual world in which he commanded on Little Round Top. A case can be made that his action prevented Confederate victory at the Battle of Gettysburg, in which case the local trajectory spreading is very great. In 1931 the Right Honorable Winston S. Churchill, viewing a hypothetical world in which Lee *had* won, described a socialist Britain, an Americanized Mexico, and a Continental Europe on the brink of unification under Wilhelm II.¹⁵

14. *Ibid.*, 94-95, from an experiment by the economist Richard Quandt.

15. Winston S. Churchill, "If Lee Had Won the Battle of Gettysburg," in J. C. Squire *et al.*, *If: Or, History Rewritten* (New York, 1931), 259-284.

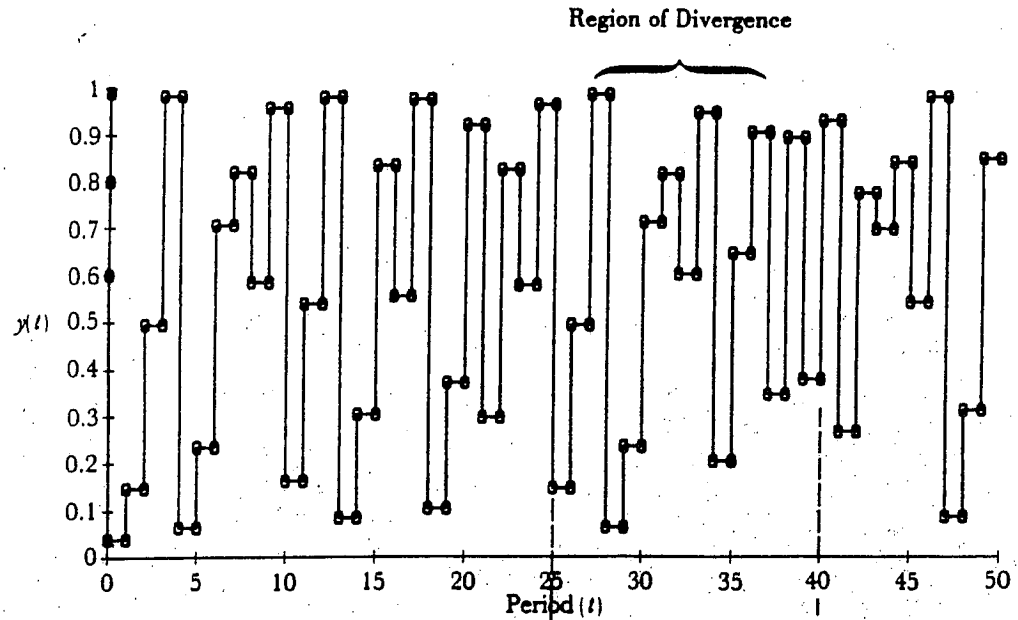


Fig. 6a. Time path, periods 0-50, $y(t+1) = 3.935y(t)[1 - y(t)]$, $y(0) = 0.99$

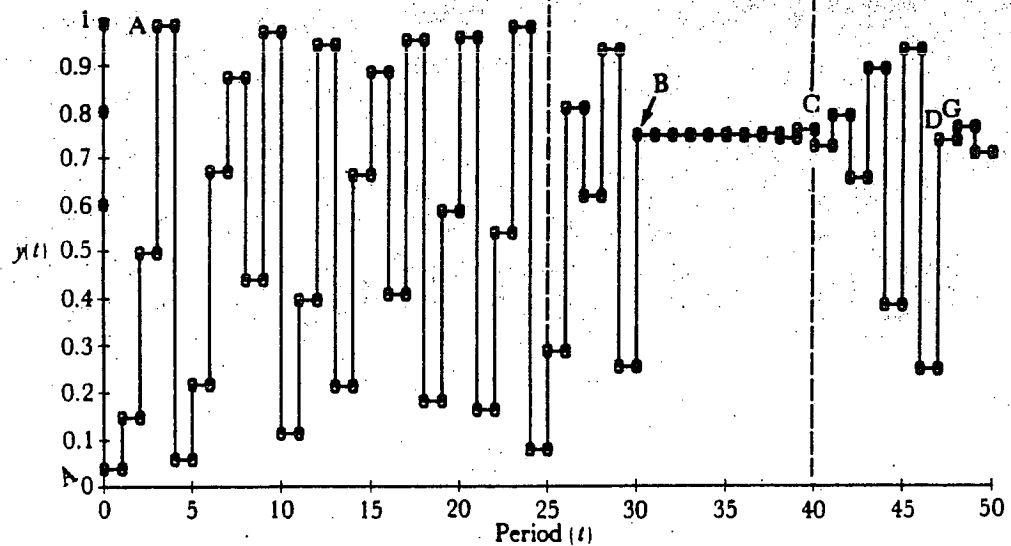


Fig. 6b. Time path, periods 0-50, $y(t+1) = 3.94y(t)[1 - y(t)]$, $y(0) = 0.99$

Figure 2

Nine caroms of a billiard ball are enough to make the gravitational field of a spectator in the room significant to the shot.¹⁶ The meteorologist Edward Lorenz, who is responsible for the notion, called it the butterfly effect. The flapping of a butterfly's wings in China can eventually produce a hurricane in Jamaica. In

16. Ivar Ekeland, *Mathematics and the Unexpected* (Chicago and London, 1988), 68.

nonlinear systems it is possible for extremely small horseshoe nails to have extremely large effects.

People are willing to believe such models because they see butterflies in their own lives. A fluttering impulse suggested to three young men on a summer's day in 1962 that the porch of Alpha Theta would be a better place to finish the next beer, just in time to hail three young women who just happened to be passing by at the time, one of whom became later one of the young men's wife of twenty-five years. The history of a nation, people reason, cannot be so different. In truth it is hard to deny.

But the attraction of the chaotic is also the attraction of magic. The accident has the power of magic, a childish omnipotence of thought in which I can change the world with a word. Each stage of mathematical education begins with magical surprises: it was surprising to most of us at 10 years old that 7 times 8 equals 56; at 14 that a quadratic equation in algebra has a formulaic solution; at 16 that an angle can be bisected with compass and straight edge; at 17 that sines and tangents are connected numerically; at 19 (an emotional high point of any mathematical education) that Kepler's Laws of Motion can be derived from Newton's laws of motion; at 21 that $e \exp i(\pi) + 1 = 0$.

At first such propositions have the arbitrary character of magic. Anyone who does not see in these an image of mysterious wisdom won by toil is intellectually dead. Tiny errors in a magical ceremony can make it go wrong. "If the Hindu magicians are to be believed, some of their rites could be practiced successfully only once every forty-five years."¹⁷ Naturally: if magic could be done on any day, in any place, it would not have the scarcity that protects its claim of efficacy. It would merely be engineering.

Education, though, consists of demystifying the wonders and making them into non-scarce knowledge suitable for engineering. Years of training in the human and physical sciences will finally drive out the belief in magic. An historian will stop telling history in the style of popular military history and strive for social significance (our freshmen are properly groping for such generality when they write "In France, during the Revolution, you were under tremendous pressure to be revolutionary"; or "The industrial revolution wasn't totally good because the machines that were invented sometimes cut off fingers and hands"; or "France is considered an isolated country. They considered themselves as Iowans do."). An engineer at some point during a long education will stop looking on a physical system as incomprehensibly organic, and will start believing that every system can be broken down into known pieces, as in the immortal paper, "Stress Analysis of a Strapless Evening Gown." He has started to think like an engineer.

The common opinion of those educated in a rhetoric of linear differential equations is that large results must have large causes. Certainly in economics the opinion is powerful. I have made a living for twenty years retailing it, attacking again and again the notion that little causes in economic history can have large effects. Most of my book reviews draw on the opinion at least once.

17. Marcel Mauss, *A General Theory of Magic* [1902-1903] (New York, 1972), 46.

But the common opinion is merely a rhetorical dogma, a way of arguing not often reflected upon, identical to the dogma of social causation in history. It can be called The Dogma of Large-Large. Large results, it says, must have large causes. A.-A. Cournot, a French savant of the nineteenth century (not irrelevantly, one of the inventors of mathematical economics, well-trained in the mathematics of engineering) wrote in 1875:

But philosophic history, the great history, concerns itself little with these microscopic causes. It seeks a sufficient reason for great events, that is to say, *a reason the importance of which is proportionate to the importance of the events*. . . . [T]he philosophic historian . . . will leave as pasture-ground for a frivolous curiosity those boudoir facts which are in themselves insignificant but which figure in the chain of causes and which we must assign to the realm of chance.¹⁸

The fall of American slavery must depend on large motives of profit, not individual morality. Capitalism must arise from irresistible social forces. A large swing of a pendulum must arise from a large push. In linear models the doctrine is true. But it can be radically false in the parts of the world that are nonlinear.

The Dogma of Large-Large, by the way, is not particular to quantification. What one admires in Marx or Tocqueville is precisely their insight into the large causes of large events, quite without mention of differential equations. By 1851, wrote Marx,

[t]he roots that small-holding property struck in French soil [had] deprived feudalism of all nutriment. . . . [I]n the course of the nineteenth century . . . aristocratic landed property was replaced by bourgeois capital. . . . The condition of the French peasantry provides us with the answer to the riddle of the general elections of December 20 and 21. . . . Manifestly the bourgeoisie had now no choice but to elect Bonaparte.¹⁹

Tocqueville's chapter titles leave little to chance: "Part 2. Chapter 2. How administrative centralization was an institution of the old régime and not, as is often thought, a creation of the Revolution or the Napoleonic period"; "Pt. 2. Chp. 9. How, though in many respects so similar, the French were split up more than ever before into small, isolated, self-regarding groups"; "Pt. 3. Chp. 8. How, given the facts set forth in the preceding chapters, the Revolution was a foregone conclusion." Such Large-Large metaphors and stories satisfy an intellect trained in history or engineering more than does the hero-worship of Carlyle or the national epic of Michelet.

Chaos pleases us, then, by reintroducing a sense of magic, a sense of many possibilities. Chaotic motion is to be distinguished from randomness. Big randomness in models of the economy leads to fatalism. Chaos — which is to say, very strong effects generated wholly within the model, but giving random-looking results — can lead to activism. The president hoping that his jawboning will end a depression is a nonlinear dynamist: he thinks that little actions of his own can

18. S. W. Floss, *An Outline of the Philosophy of Antoine-Augustin Cournot*. Ph.D. dissertation (Philadelphia, 1941), 99–100, italics added.

19. Karl Marx, "The Eighteenth Brumaire of Louis Napoleon" [1852], in Lewis S. Feuer, *Marx and Engels: Basic Writings on Politics and Philosophy* (New York, 1959), 341–342, 345.

overwhelm the natural randomness.²⁰ Or alternatively a grasp of chaos can lead to proper caution: small lags in pushing brakes or the accelerator can be disastrous on Route 80, and on the troubled route of economic policy, too.²¹

The economists William Baumol and Jess Benhabib note that in regions of chaos "a time path is sometimes extremely sensitive to microscopic changes in the values of the parameters—a change in, say, the fifth decimal place of one parameter can completely transform the fifth qualitative character or the path."²² We do not know the fifth digit (much less the fifth decimal point) of most statistics. To achieve such accuracy the 1990 Census in the United States would have to be accurate to plus or minus 10,000 people (which it is not difficult to imagine as the number of homeless or illegals uncounted in New York City alone). As Oskar Morgenstern wrote in 1963, in an unsuccessful attempt to persuade economists to adopt the error-conscious precision of engineers and physicists, "[s]tatements concerning month-to-month changes in the growth rate of the nation are nothing but absurd."²³

The butterfly can take flight either in the parameters (that is, in the confidence about the model imposed) or in the initial conditions (that is, in the confidence about the observations of the world's condition). Both yield large differences out of small differences. Only unreasonable dogmatism about the model or unreasonable dogmatism about acuity can restore one's confidence in the Dogma of Large-Large.

The problem (and this is the main point) is that in nonlinear parts of the world the idea of storytelling is cast into doubt. "This is why long-range weather forecasting is so difficult: everything, absolutely everything, must be taken into account."²⁴ American history 1856–1865, in the opinion of two careful students, was in such a precarious state that small events could have a big effect. The rogue sheriff and the bold professor of rhetoric "changed history," as we say.

But in that case *any* of an unbounded set of little people and little events could be brought into the story. Unknown to history, a certain John Jones in Kansas, who alone had the moral authority to stop the sheriff, failed to arrive in the posse (he had a bad cold and was in bed). Likewise unknown to history, a political general named Robert Smith in 1861 had assigned Chamberlain to the twentieth Maine quite by accident—Chamberlain should have been put in the second, not the twentieth, but it was late at night when Smith did the job, and the orders had to go out by the next morning, leaving no time to check them. George Burns said on his TV show once, "George S. Kaufman is responsible for the skit to-night: I asked him to write it, he refused, and so I had to do it."

20. William A. Brock, "Introduction to Chaos and Other Aspects of Nonlinearity" [1987]. Working paper, Department of Economics, University of Wisconsin. Published as Chapter 10 in *Differential Equations, Stability, and Chaos in Dynamic Economics*, ed. W. A. Brock and A. G. Malliaris (New York, 1988), 9.

21. Baumol and Benhabib, 80.

22. *Ibid.*, 79.

23. Oskar Morgenstern, *On the Accuracy of Economic Observations*, 2nd ed. (Princeton, 1963), 304.

24. Ekeland, 66, speaking of Lorenz.

In some counterfactual world the Civil War and its outcome might have been governed by big, simple, linear metaphors — slavery might have been steadily less profitable despite southern sentiments, the North might have been destined to win despite southern generalship. The success of Christianity, likewise, depended on the Roman Empire, and the Industrial Revolution on the freedoms of northwestern Europe. But if, as Fogel and McPherson and many historians before them have persuasively argued, the correct models for 1856–1865 are models of nonlinear feedback then the story becomes unmanageable, untellable. It is a paradox, beyond the common opinion of Large-Large.

What we can do is look for times that seem chaotic and be forewarned. That is what engineers do. In regions or times that seem chaotic they note the pattern of onset but do not otherwise try to predict the motion of the swirling water.

Are we in a nonlinear and even chaotic world and if so what can be done to go on telling stories in it? I have already discussed one way of knowing whether we face chaotic systems, namely, by building up a model from horseshoe nail to kingdom. Engineers and ecologists use the technique, starting with a nonlinear equation in which they have acquired confidence and noting that for observed values of its parameters it does indeed imply chaotic behavior.

If one already knows the laws of motion then the strategy of simulation (which is what it is called) is fine. But it has difficulties for historical narration, where what we know of a metaphorical and model-building sort does not come usually with laboratory-fitted magnitudes.

The alternative to simulation as a method of reasoning in science is look-see. You can either build a model of a star on the blackboard from first principles and see how the model behaves or you can go up the mountain to the telescope. Whether or not you have a believable and numerical model of evolution you can merely look at the results and see. But the look-see evidence for chaos is sometimes hard to assess. The economists who doubt that chaos has much to do with the macroeconomic features of the economy (growth rates, unemployment, inflation, and the like) point to the stability of certain magnitudes, such as the saving rate (when correctly measured) and the real interest rate; and they point to the tendency of the economy to return to its trend after being disturbed.²⁵ Fair enough: if the economy does not exhibit erratic and nonrandom motion like that in Quandt's second experiment above then all may be well. But the economy sometimes *does* exhibit such motion, as most famously in the Great Depression of 1929–1939. Is it chaos?

The most direct way to untangle the chaos and to make look-see work is to guess, by God's grace, what the order of the differential equation is. The "order" is how many derivatives it has, that is, how many rates of change of rates of change. The first-order *linear* differential equation of the simplest sort marches up by 80 million a year. Plotting its t^{th} value again against its $t + 1^{\text{th}}$ will show a straight line. If the *nonlinear* equation is of first order, similarly, you can plot the successive points on a graph of x_t and x_{t+1} . The nonlinear hump will appear out of

25. Brock, "Introduction to Chaos," 7–8.

the gloom (look at the heavy dots in Figure 1 above, tracing out the hump), and the same is true of higher orders in the underlying differential equation. The feature is called "dimension," or as the mathematical economist William Brock puts it, "the minimal number of lags of x that one would need to describe the dynamic behavior of $x(t)$ in the long run."²⁶ For a system that God has told you has only one lag (the first-order system, in which x_{t+1} can be fully determined by knowing x_t) the dimension is officially 1, and a two-dimensional diagram plotting x_t and x_{t+1} achieved is enough to fully reveal the structure.

The trouble is that you have to have God's grace. You have to guess what the lags are in the world's equation. As Baumol and Benhabib point out, "this problem is no different from that in choosing the structure of a model for econometric estimation," that is, the usual statistical problem.²⁷ Attempts to solve the problem without recourse to rhetoric (that is, by absolute and nonhuman standards) have driven econometricians insane, because the problem of choosing the model is that of choosing a human point of view. It is something done by us, not by God, and is therefore not absolute and not nonhuman. One is going to be driven insane if one tries to find a nonhuman point of view from within a hopelessly human problem. (The rhetoric of the scientific conversation, I have argued elsewhere, provides the practical solution, and keeps one's sanity.)

The problem is that a simple model with horse-kick randomness looks much like a wholly deterministic but chaotic model. You often cannot tell the difference between old-fashioned randomness—which, God knows, is prevalent in this life—and new-fashioned chaos. Chaotic randomness is often jerky: it looks random and deterministic by turns, exhibiting what economists call "régime changes" from time to (unpredictable) time. Some types of randomness, though, could look jerky, too.

Low dimension chaos is the only kind worth worrying about, because of the difficulties of measuring anything without error. "Low dimensional chaos" means essentially a return to the neighborhood of a given point after a certain number of periods—high dimensional chaos would be indistinguishable from randomness (randomness is infinite dimensional chaos, so to speak²⁸). Well short of high dimensional chaos the cycles in a nonlinear system become so mind-boggling that in practical terms they cannot be known. Only in a system without appreciable error of measurement or appreciable pure randomness could a mathematical technique extract them infallibly; the alternatives are statistical techniques, but short of experimental perfection they always face some probability of false positives. Chaos, technically speaking, is a limiting case, in which the pattern of repeating values has, so to speak, an infinite number of cycles, with the effect that no point is exactly repeated, ever. Parts of the chaotic regions will show patterns, with the points clustering. In other regions the Small-Large will hold.

26. William A. Brock, "Chaos and Complexity in Economic and Financial Science," in *Acting under Uncertainty: Multidisciplinary Conceptions*, ed. George M. Furstenberg (Boston, 1989), 424.

27. Baumol and Benhabib, 101.

28. See Brock, "Introduction to Chaos," 8.

The ambition to spot low-dimension nonlinearity faces, however, an impossibility theorem. It stops the narrative again. Brock points out that in stock markets a low dimension chaos would be searched out and used to make money.²⁹ The chartists and other technical elves in the stock market, he notes, are essentially claiming to detect recurring patterns by suitable choice of lags. But the claim is unbelievable. If they were so smart they would be rich.

The criticism is devastating to any pretensions to see simple patterns in history. I would add only that more than stock markets have this problem of claiming (unbelievably) to know how a differential equation works. If the patterns of chaos were so simple then the actors in the history would see them, and would eliminate them by making use of their knowledge. Historians need to discipline their stories by making sure that they do imply such stupidity as even Haig or Louis XVI would see through.

The problem of chaos, then, is the problem of unpredictable and inexplicable behavior. Such behavior does not arise simply because God plays dice but because we mortals cannot monitor His/Her dice playing closely enough, in nonlinear cases, to predict even the larger events. The problem is intrinsic to narrating human life. The literary critic Peter Brooks observes that "our lives are ceaselessly intertwined with narrative, with the stories that we tell, all of which are reworked in that story of our own lives that we tell to ourselves. . . . We are immersed in narrative."³⁰ We have met the "we," however, and they are us. The fossils do not tell a story about the Cambrian explosion: we tell it. For the human sciences the case is worse. The water molecules in even a turbulent flow are not listening to the story, and for all the principles of least action they are not telling stories about their own lives.

Perhaps chaos is merely the historian's way of thinking getting into science. It is a new way of arguing in economics and other mathematical fields, but in the end it comes down to Cleopatra's nose: if she had had a different nose, unattractive to Roman generals, the battle of Actium might not have happened,³¹ and that could have made all the difference. Narration in a nonlinear world is difficult regardless of whether the problem is numerical or not. One does not avoid nonlinearities by not knowing what they are called. When success breeds success, when variables feed back into themselves, we have an exciting story to tell, but unless we know its metaphors already we have no way to tell it.

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29. Brock, "Chaos and Complexity," 427.

30. Peter Brooks, *Reading for the Plot: Design and Intention in Narrative* (New York, 1985), 8.

31. J. B. Bury, "Cleopatra's Nose," in *Selected Essays of J. B. Bury*, ed. Harold Temperley (Cambridge, Eng., 1930).